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TOWARD A MATHEMATICAL MODELING OF CREATIVITY

"Creativity" is fascinating! We know so much about the topic without having the slightest idea what it is. We even know how to promote creative behavior by encouraging subjects to avoid strict adherence to rules, to explore new explanations or paradigms, to welcome novelty, to transform a problem into a different framework, to generalize through abstraction, to brainstorm, and to explore options before making evaluations. But, other than knowing that these approaches tend to work (sufficiently well so that some are "corporate brainstorming strategies" while others are tried in the classroom), we really don't know why. The "why" becomes an important issue. Also, all these different approaches emphasize "generality" over details. What explains this commonality?

Continuing, researchers have uncovered clues suggesting that creativity is associated with a complex dynamic. This is manifested by those "thinking is not thought" phrases. The purpose of these catchy comments is to focus on the immense gap separating where conscious and dedicated acts are used to resolve a cognitive conflict from some sort of sub- or preconscious rearrangement of ideas. This subconscious notion includes the important and not uncommon "aha!" experience where a difficult problem suddenly is resolved during a period of rest, upon awaking, or while taking a walk rather than during a period of active work. Well known examples of these almost religious happenings are described in Mozart's famous letter, in Poincaré's often quoted lecture, and in Hadamard's book (1945). This also is part of Helmholtz's "inspiration" stage that Poincaré includes in his outline of creative thinking.

(The Helmholtz and Poincaré stages have been rediscovered by many others.) As this preconscious activity is an important creativity characteristic, we need to understand why and how it happens.

"Triggering", where seemingly unrelated events unleash an idea, is another fascinating creativity phenomenon. A red car driving past may trigger a solution to a financial problem; overhearing a casual conversation may trigger, in mysterious ways, the resolution for a difficult scientific problem. Why?

The mystery continues. When pressed for an explanation of creativity, standard responses invoke "intuition". But, what is intuition? Typical answers

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governmental policies frustrate, rather than assist, the development of a creative society. (To illustrate, consider the continual conflict between the need for expression and newly imposed standards in our schools. When forced to prepare for new tests and curricula, teachers occasionally are criticized that their students are not as creative.)

This essay describes portions of a more general model about cognitive processes. The selected traits (from an extension of (Saari, 1978)) provide plausible explanations for the “aha” experience, intuition, similarities in “creativity enhancing approaches”, and “triggering”. Because our assumptions are governed by the mathematics, not cognition, we are encouraged that several conclusions resemble common behavior. But, to keep our exposition from becoming overly abstract, intuitive descriptions replace a careful, formal presentation. Appropriate terms, however, are defined in the footnotes so that the reader with a mathematical background can supply the missing details of our plausibility arguments.

TOWARD MODELING

Our model is simple and sparse; it uses minimal, accepted assumptions about thought processes. Essentially, we just assume that, somehow, information is conveyed from the outside world to the attention of the individual, the organization, or the computer. From this barest of structures—the properties of transmitting information—we extract relationships and consequences. By using standard assumptions, our results extend to (but may not have been noticed) other models.

As stated, our assumptions are not motivated by the cognition or creativity literature; they are dictated by the mathematics. At each stage, the mathematics requires selecting from several options; we explore and describe the consequences of certain paths. So, those abstract conclusions which resemble creative and/or cognitive properties identify which assumptions explain such activities for people, organizations, or computers. Much of our discussion describes parallels between the abstract conclusions and results from the cognitive, creativity, and learning disability literatures.¹

We emphasize adaptation—a central, widely accepted concept in psychology used to model “adjustment”. Our objective is to find a “cognitive adaptation” foundation to unite and explain a wide variety of seemingly disparate creative and cognitive behavior. In this way, we determine which of the tacit assumptions of this field already admit creativity-like behavior. As the modelling involves the adjustment of information and as similar adjustments occur in society, organizations, and even computer programs, there exist a wide spectrum of applications and ways to test our conclusions.

¹ We view comparisons with the cognitive and the LD literatures as a partial test of our assumptions.
Adaptation has to occur somewhere, so the mathematics requires specifying the underlying space. Treat this space as a sterile “black box” that could represent a person, an organization, a computer, or whatever is being analyzed. Differences in derived consequences depend upon the properties of each black box.

World views

There are many novel ways the “world” can be viewed; our model must admit all of them. This is important; if creative people view the world differently than us more mundane beings, then their insights must be represented. In fact, this issue needs to be confronted if we hope to address the concern that “many people assume that there will never be a scientific theory of creativity—for how could science possibly explain fundamental novelties?” (Boden, 1994, p. 75) Fortunately, the mathematics demands a wide variety of differences. As these “world-views” form the foundation for the more interesting “creativity” discussion, we devote considerable attention to this issue.

Piaget does not state the “world view” issue in the above manner, but he does emphasize the central nature of the problem by labelling the way a person interprets the world as her “Organization”; this “Organization” is important for Piaget’s theory. This term with a similar meaning appears throughout the literature. For instance, with Krash and Smith’s (1970, p. 27) approach which places a heavy emphasis on methodology, “[t]he concept of organization will denote the visual form reported by the subject verbally and/or in the form of a drawing”.

Following standard usage “environment” refers to the outside world. The environment’s actual organization (e.g., behavior determined by physical laws, gravity, or imposed by “our” interpretation), however, need not agree with a person’s Organization. “Correctness of interpretation” may be central for creative product approaches and needed in a lab to distinguish between clever and nonsensical insights of an experimental subject, but it is counter to an “abstract” study of creativity because it imposes assumptions before they are needed. (Insisting on accuracy requires imposing an all-knowing being to determine “correct answers”. But, if creativity includes an ability to develop new paradigms, then creative interpretations differ from accepted views. So, what is “correct”? “As correctness” changes, one must be suspicious of models requiring agreement between a person’s and “our” Organizations. Instead, conflict (as defined by the individual) should drive the creative process. Again, this is consistent with the literature; Erickson, for instance, relates a person’s development with his interaction with society.)

Understanding occurs somewhere, so treat the center of “thought processes” as a non-descript “black box”, denoted by BB, where information from the world is conducted to BB through input nodes. This just resembles the standard computer metaphor for information processing. Our refinement comes from examining the consequences of encoding information.

So far BB only possesses input nodes to convey information from the outside world. If these nodes represent, for instance, our senses, then one node might transmit visual information, another might convey sound, still another might register taste. Should BB represent a computer, the input nodes have a standard “hard wire” interpretation. When BB models an organization, each participant in the information and/or decision processes serves as a node. We need to determine how different ways to transmit information affect understanding.

Interpretation

Borrowing from Piaget, “understanding” an event is characterized by how an individual organizes or arranges the environment. As indicated, Piaget calls this structuring “Organization”. In our setting, the input nodes convey information or stimuli from the environment E. The definition of E is determined by (and can change with) the choice of BB and the input nodes. For instance, while E need not include a Saturday football game if BB is a computer, it can when BB represents an individual. To repeat, the structure of E is essentially ignored. (How the E structure affects “feedback” is an important part of the more general model that is not described here.)

Instead of allowing the consequence of what is transmitted to rattle around BB, assume each input, or event e, has a unique output in BB. By treating the jth node as a mapping

\[ f_j : E \rightarrow BB, \]

a natural definition for BB’s “understanding” of event \( e \in E \) is the image \( f_j(e) \in BB \).

This functional relationship captures minimal aspects of most, if not all, theories of cognitive development. After all, it just asserts that, somehow, information is brought to BB’s attention. For instance, Eq.1 corresponds to the input node of the computer metaphor; it represents basic aspects of Piaget’s “assimilation”, and it captures minimal parts of Smith’s “Construction in the direction of the object” stage in his Precept-Genesis (PG) analysis (see (Smith and Carlsson, Fig. 2.1, p. 17)). For other illustrations, one node for a mathematician may correspond to an analytic representation of a problem, while another may be a geometric formulation. More loosely, think of an “interpretation” (the image of the mappings) as the retrieval and association of a current event with past experiences.

Already mathematics poses questions to explore. For instance, most mappings identify many inputs with the same image.\(^2\) For Eq.1, this requires several events to share the same interpretation. This, of course, captures the

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\(^2\) Generically, functions are not one-to-one.
human behavior where the same meaning can be associated with each of
"peculiar", "peculiar", "peculiar", "PECULIAR", or "peculair"
even though the second and last choices are misspelled and different typesetting
is used. So, an immediate abstract consequence of Eq. 1 corresponds to a
well recognized cognitive trait. (In fact, a curiosity is how conclusions from
most mathematical assumptions admit illustrating examples.) In highly special-
ized situations, mathematics allows input nodes to have the property
where each $b \in BB$ represents a unique input $e \in E$. If this property models
a computer programed to recognize only special fonts and correct spelling,
then only one "peculiar" choice may be accepted. As we show later, abstract
arguments suggest that this precision has a cost; intuition is lost.

For another implication of Eq. 1, if two input nodes differ, even very
slightly, then they have different interpretations for at least one input. Again,
this property has close parallels with human and organizational behavior;
e.g., we know that a variety of events can be equated with hearing something
while a different collection might be equated with seeing it. Carrying this
comparision of different functions a step further, it allows individuals—even
with similar wiring—to have different world views. After all, difference in
the nodes requires events which lead to different interpretations. Thus, just
by assuming Eq. 1, the model permits even seemingly similar individuals or
organizations to exhibit different Organizations; this meets our goal of
admitting a wide spectrum of Organizations.

Organization

To extract further Eq. 1 consequences, observe that BB's "Organization" are
all events sharing the same interpretation; this is for all choices of $b$ in BB.
Thus BB's Organization critically depends upon the choice and properties of
how information is transmitted. To mathematically define "Organization",
recall that the inverse function $f^{-1}(b)$ identifies all possible inputs—all possi-
ble events — with $b$ as the output (the interpretation). As $f^{-1}(e)$ is the BB
interpretation of $e$, $f^{-1}(f(e))$ is the set of all events sharing the same interpre-
tation as "$e$". This is illustrated in Fig. 1 where the interpretation of event, $e$,
is depicted by the arrow pointing to the right, $f(e) = b$. The two dashed
arrows identify other events from the environment, $f^{-1}(f(e)) \in E$, with the
same interpretation; i.e., $f$ cannot distinguish among them. (Compare this
statement with the figure and the "peculiar" example.) "Organization", then,
is the structuring of the environment as determined by these inverse sets.

1 The mappings are one-to-one.


Figure 1. Organization; events with $f(e) = b$ interpretation

This Eq.1 modeling requires an "interpretation" (the image element $b$ from
BB) to be based on the properties of the entries of BB. If, for instance, BB
models a computer that just lists physical objects, identification is the only
allowed interpretation. However, should the BB modeling admit relations-
ships, then $b$ could be an admissible relationship involving, perhaps, addi-
tion or multiplication. If, for instance, $e$ is the event of tossing three apples
in with four apples then $f(e) = b$ might be the abstract "$3+4=7$" addition
process and $f^{-1}(f(e))$ might represent other "$3+4$" addition problems involv-
ing cars, or strawberries, or ... 4 Then again, the transmission of information
(the input node) might only associate fruit with addition. Namely, it follows
from the mathematics that determining whether events can be interpreted,
how they are interpreted, and how they are associated with other events
critically depends upon properties of the input nodes and the BB structure.
Notice the variety of world views! So, one way we introduce a variety of
ways to view $E$ is by including all ways to transmit information. Compare
this inclusive approach with the tendency—almost an obsession—of other
models to specify exact properties. This specification, or "fixation", imposes
unnecessary obstacles to understanding creativity.

Creativity

This "different people view the world differently" structure sheds light on a
mystery of the creative process. We have in mind Hadamard’s (1945, p. 49)
worry about "the failure of a research scholar to perceive an important
immediate consequence of his own conclusions". How could this be? How
could a bright person who is the world expert on a new concept by discov-
ering it, miss an important, "obvious" consequence? As Hadamard shows
with examples, this is not uncommon.

A plausible explanation uses the observation that different functions have
different inverse sets. So, treat a new discovery as an input $e$. For me, $e$ is
associated with all events $f^{-1}(f(e))$ for you and your input node $g$, the same

4 Nothing in this modeling requires awareness of all possibilities. For instance, we don’t know all events that can be combined, but if they involve addition, we still would use arithmetic.
event is associated with all events \( g^{-1}(g(e)) \). Differences in how we view the world, as represented by different input nodes, require us to have different associations with the same \( e \). So, even if I understand the idea I invented better than anyone else, our differences of associations, modeled by each of us having different input nodes, allows you to make associations with \( e \) that I failed to see; you could find implications I missed. A similar description holds for organizations. IBM started the personal computer revolution in the early 1980’s and then was almost undone by it because it did not grasp the impact of what it had set in motion. When customers began to abandon their expensive IBM mainframes for clusters of cheaper personal computers, Big Blue was unprepared. Its revenues plunged, . . . .2

What does this abstract structure suggest about developing creativity? The most obvious lesson is to avoid adopting identical world views. In order to generate different associations — and potentially different insights — the structure of \( BB \) and/or the input nodes must differ. In terms of a person or organization, this suggests avoiding becoming a carbon copy of someone else. Indeed, successfully mimicking someone is modeled by assuming both have essentially the same Eq. 1 functions. As the inverse sets — the associations — closely agree, it becomes difficult for an imitator to discover something new through different associations.

In practical terms, this supports J.E. Littlewood’s advice “[d]on’t think you must read up all the literature that it might have a bearing [on a new and difficult mathematical problem]”. To develop a creative approach, Littlewood (1968) advises that “there is much to be said for going ahead . . . without reading anything beyond . . . the minimum to find what the problem is about”. Littlewood’s suggestion is consistent with what is suggested by our abstract structure. Remember, the input nodes represent how information is transmitted. So, a researcher carefully reading the literature — beyond understanding the problem — runs the risk of representing, formulating, and transmitting information about the problem in a way similar to the giants in the field. Even children know better than to compete with giants on their terrain with their weapons.

Simple examples

The assertion that radically different Organizations emerge with different choices of \( BB \) and/or how information is transmitted (the input nodes) is so critical that it is worth illustrating how the conclusion comes from the model rather than an imposed wish. To do so, we invent easily analyzed “black boxes” which have nothing to do with humans or organizations.

Our test site has \( E \) as the set of all real numbers — positive, negative, and zero — and \( BB \) as the set of non-negative numbers. One mapping to convey

information is \( g(x) = |x| \); it interprets the world of numbers, \( E \), in terms of a number’s magnitude as \( g \) ignores the sign. This “input node”, therefore, cannot distinguish between \(-1 \) and \( 1 \); i.e., \( g^{-1}(g(1)) = g^{-1}(1) = \{-1, 1\} \). Thus the world according to \( g \) (i.e., \( g \)'s Organization) combines numbers into pairs according to magnitude. (Compare this description with Fig. 1.) Contrast \( g \)'s world view with that of the mapping \( h \) which drops the integer portion of a number while retaining the fractional part. As \( h \)'s Organization is defined strictly by fractional parts, this input node prohibits \( BB \) from distinguishing between \( 4.5 \) and \( 6.5 \). Indeed, as \( h(4.5) = .5 \), it follows that \( h^{-1}(h(4.5)) = h^{-1}(0.5) \) is the set of all numbers with fractional part \( 0.5 \). So, \( g \) and \( h \) have the same interpretation of \( 0.5 \), but \( h \) detects no difference between \( 0.5 \) and \( 100.5 \) while \( g \) finds a major difference. Notice how differences in transmitting (formulating, etc.) information can significantly alter interpretations.

To show that even slight differences in input nodes can generate major differences in perception, treat \( h \) as the remainder of a number when divided by \( 1 \). A closely related choice is the function \( k \) which specifies the remainder of a number when divided by \( 1.1 \). While \( h \) and \( k \) agree on the interpretation of the input \( 0.99 \), they disagree about \( 1.01 \); \( h \) views \( 1.01 \) as the insignificant \( 0.01 \) while \( k \) interprets it as the large value \( 1.01 \). Thus, even slight changes in input nodes can create significant differences in interpretations!

This mathematical example underscores the sensitive dependency of the Organization, or world view, upon the properties of the input nodes. With the same \( BB \), radical differences in the world views of \( g, h, \) and \( k \) result from the different ways they convey information. This general consequence of Eq. 1 occurs in all settings.

As for creativity, this structure suggests that imaginative outcomes need not always be the result of personality, drive, or smarts. It suggests that even a seemingly dull person can, at times, appear to be imaginative. Again, even closely related actions admit different Organizations. So, while different individuals or organizations usually have essentially equivalent interpretations, any slight differences could allow situations with very different interpretations! While such differences may correspond to bankruptcy or another disaster, they also might allow a person identified with mundane views to have a creative interpretation in certain settings. Is this creativity? Is it if the outcome is a new view. So, this statement, which depends on what associations are made, suggests that some types of “creativity” may be due to circumstances. (The movie Forest Gump cleverly creates settings where an undistinguished person excels.)

Similarity

Once an Eq. 1 relationship is specified, the next mathematical step is to worry about the “topology” of the image space. This requires determining
whether and how BB accepts two interpretations as being almost the same. To illustrate, our BB consisting of non-negative numbers defines two numbers as being close or similar when their difference is "small". The "peculiar" example includes the slightly misspelled words.

By adding this similarity structure to BB, the "Organization of E" inherits a sense of "closeness"; inputs are similar should they admit "nearby" BB interpretations. In other words, the mathematics provides a working definition for "associated events" — a key concept from cognition and creativity.

Illustrating with our mathematical examples, the input node g treats -5 and -5.01 as being essentially the same because g's respective interpretations of 5 and 5.01 differ only by a tenth of a unit. While this seems obvious, remember that "closeness" is determined by how information is transmitted. To illustrate, while we don't accept -50 and 50.0001 as being near each other — they differ by more than a hundred units — g does. By interpreting these events, respectively, as 50 and 50.0001, g finds them to be closer together than -5 and -5.01. This example underscores the (Eq. 1) fact that interpretation is in the eye of the beholder.

Notice how this example illustrates our earlier comment that BB's Organization of the universe need not be accurate, nor reflect what someone else accepts. Instead, it is influenced by the input node. For instance, while g judges 2.3 to be closer to 2.1 than to 9.11, h disagrees. Because h interprets 2.1, 9.11, 2.3 as being, respectively, 0.1, 0.11, 0.3, h treats 9.11 as being closer to 2.1 than to 2.3.

The assumption that the BB structure defines a sense of "closeness" or proximity provides an even richer variety of "Organization" structures. Differences in the assumed BB structures (e.g., different educations, cultural backgrounds, or upbring), introduce even more "world views". This makes sense; it captures the sense that different life experiences can cause different interpretations of events. (To illustrate with the test site of BB and function ShS, let "closeness" in BB now be defined by the remainders when a number is divided by 10. This forces 9.99 and 10.01 to be far apart because the first has a remainder of 9.99 while the second has a remainder of 0.01. Therefore, the new BB structure combined with k now renders 9.99 and 10.01 as being distance, while the former structure had them close to each other.)

Thus, changes in the structure of BB and/or how information is transmitted can alter BB's world view. This preliminary portion of the modeling already quantifies and explains those "People think differently because they are different" type of statements. In particular, it now is easy to extend the earlier "creativity" comments concerning Hadamard's concern about why an inventor of an idea need not recognize important applications. Different life experiences define different definitions of "proximity". In turn, this can create radically different associations and Organizations. Consequently, we

* Thus "E has the weak topology induced by \(\{g_j\}\)."

TOWARD A MATHEMATICAL MODELING OF CREATIVITY

must expect the same stimulating event to have very different meanings and associations with different people.

Aggregated interpretations

Different BB input nodes create conflicts in interpretations. This is because each mapping defines a preferred Organization of E, and, by being different, these Organizations cannot always agree. This conflict needs to be resolved.

Nonabstract examples of this conflict are easy to find. It already occurs with our sensory input nodes where one is hearing and the other is sight. "Seeing" a small portion of a red vehicle, for instance, might generate a wide equivalency class (that is, a large number of possible interpretations and associations) ranging from a flashy sports car to a fire truck. Hearing a siren brings forth interpretations ranging from a police to an ambulance to a fire truck. The final interpretation resulting from aggregating information gathered by these senses is obvious; it is a fire truck simply because that event is common for both mappings.

Figure 2. Reaching an interpretation

Similarly, each Fig. 2 oval depicts the Organizations generated by an input with each way to transmit information. Define the BB aggregated interpretation to be the part common to all of the input mechanisms; it is the intersection (the heavier shaded region) of the different different BBs. As an intersection includes only what is common among the Organizations of the different nodes, it represents what is understood without conflict.

To show how this definition resembles behavior in organizations, computer, and humans, start with the fact that an intersection is an exclusionary act. Thus, the aggregated interpretation can be thought of as the end result of some sort of evaluation process. While the dynamic is unspecified (Sect. 4), it has the flavor of evaluating through comparing the associations of all the input nodes. Thus the aggregated interpretation — the intersection — represents a completed evaluation.

To better appreciate this notion, suppose BB, normally serviced by several
mappings, finds some nodes inoperative. Figure 2 dictates what must happen. As some sets (ovals) are missing, the intersection is larger. This carries the sense of a fuzzier interpretation—one that is not as refined or sharp as it would have been. This fuzziness represents the unmade comparisons from the missing nodes.

With reflection, this description is as it should be; because $BB$ is missing refinements afforded by other perspectives, the final intersection is not as carefully "evaluated", so it need not be as refined. This abstract conclusion admits immediate parallels with the problems of humans who are blind, deaf, or suffer a learning disability. In each setting, the deficiencies of a particular way to convey information dictates a gap in the Organization; comprehension is impaired. A related behavior is when an organization loses a member with a unique and valued perspective on organizational issues. For individuals or organizations, assistance is provided by finding how to overcome the roadblocks.

Notice the distinctly different meanings for the "size" of these regions. A large "size" of the events directly associated with a particular interpretation represents "fuzziness" of thought. This makes sense; it means that discriminations have not been made. On the other hand, a large size of events accepted as being similar to an event indicate a wide range of associations. These associated events can be equated with an individual who has an awareness of other data, a flexibility of thought process, an ability to fuller utilize other events from the environment—it can be associated with creativity.

So, different distinctions of the "size" of these regions could represent "creativity" (when wide and unusual associations are emphasized), or only a partially developed sense of the world (when many undistinguished events are viewed as the same) such as a child using "trucking" to identify all vehicles. While these two interpretations of a "large region" represent very different structural sources and different levels of sophistication, both require a "wide association" with a specified input. Thus we must expect "creativity" to share child-like behavior. (This connection is not original, but its explanation is.) In turn, these comments provide partial support for experts who worry whether creative-like actions in children indicates creativity, or an undeveloped sense of discrimination.

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1 The induced topology is cruder than the original one.
2 All conflict in "size" differences disappears when these ideas are formally expressed in terms of topology. "Sharper" ideas correspond to a refined topology; abundance of associations is identified with the size of open sets.

Further differences

Significantly different "Organizations" can be generated by imposing different structures on how $BB$ interprets "closeness". At one extreme, $BB$ may not have such a definition because every event is interpreted as being distinct from all others.2 This $BB$ can only cluster events that are equally interpreted; there is no ability to understand that events are related or close to one another. This can be illustrated with the early word processors on computers—until a notion of "proximity" for words was programmed into the computer, spell checkers were impossible because all words had to be interpreted as distinct entities.

Examples mimicking this abstract behavior are easy to find from, say, the learning disability literature. For instance, compare these "discrete topology" consequences with the observed "Executive function difficulties" in retardation. Here, the person can perceive various stimuli, but he cannot pay attention to relevant aspects of a problem—he cannot recognize the close relationship of other events.11

The other extreme is where "everything is close to everything else".12 Here $BB$ has different interpretations for different inputs, but "everything" is related without distinction whether something is "more similar" than something else. Immediate mimicking examples come from certain computer structures; unless programmed, the computer may be able to recognize that one string differs from another, but it clumps them all together—everything is essentially as different as anything else.

The rich variety of structures between these extremes provide a more comfortable middle ground; they allow for varying degrees of similarities. Examples resembling such behavior include the spell-checker illustration. Some spell-checkers suggest alternatives for misspelled words only if they differ by a letter; others, where the programming provides a more sophisticated notion of "closeness", offer several alternatives. Interesting related examples come from the child development literature. One example is the tendency of a child to first learn the letter "o", and then the child equates all letters with "o". You can almost see the crude sense of "closeness"; if it is a letter, it is called "o". Once differences—more discrimination—are noted, letters are called by the correct name. Notice how when the proximity regions become more refined, there is a concomitant change in the interpretations. Similarly, with the earlier "trucking" example, much too quickly the child learns to accurately identify a passing vehicle as "a '57 Chevy with its classic wings, but with four-on-the-floor and eight cylinders". 

9 That is, by imposing different topologies on $BB$.
10 $BB$ has the discrete topology.
11 So, a person without this problem is modeled with a reasonable topology; a person with it is modeled with the discrete topology where "closeness" does not exist.
12 Here $BB$ has the trivial topology.
TOWARD INTUITION

How do creative people discover original solutions? As usual answers involve “intuition”, we need to represent this sense that we can know something without knowing it. This description transforms intuition into something that is suspected but not recognized as being relevant. Our model already admits this notion.

To explain, assume several “input-output” mappings service BB. (Again, when nodes are defined by our senses, there is at least one input node for each sense.) Figure 2 is a schematic representation for the expected conflict in the Organizational regions; a conflict that becomes more pronounced when the “similar interpretations” admitted by the proximity structure of BB are used. In Fig. 3, we describe these regions in terms of intuition.

Each Fig. 3 oval represents an input node’s Organization. The heavier shaded intersection is the firm, aggregated interpretation of e. The remaining regions—depicted by the portions of the ovals outside of the “interpretation region”—represent associated events that, through at least one input node, BB treats as being related to e. What prevents these other events from being identified with e is that they are eliminated from the intersection through the evaluation processes—they are removed because of their conflict with associations admitted by the other ways to transmit information. So rather than serving as an interpretation of e, events in these regions are only “suspected” of being related to e. These oval regions outside of the intersection, then, model a form of “intuition”; call them the intuition regions.

This definition replaces the mysticism of intuition with a property that can be examined, tested, and even exploited. For instance, instead of identifying the act of discovery with mystery, luck, pure chance, or accidents, this definition anchors intuition and discovery upon previous preparation (to develop the proximity structures of BB and how information is transmitted). It suggests how to introduce and evaluate personality traits and means of “thinking” to facilitate creativity. (Certain traits and approaches better allow these intuition regions to be explored than others.) Not only does this comforting comment support the virtue of hard work, but it provides relief to Hadamard’s concern [1945, p. 19] about descriptions concerning new discoveries. He worried that “explanation by pure chance is equivalent to no explanation at all and to asserting that there are effects without cause”. Our structure, critically based on what BB knows, avoids this creativity trap.

Immediate “intuition” consequences follow from the now standard observation that different ways of transmitting information define different associations. By considering different kinds of nodes and/or different kinds of “proximity structures” on BB, different kinds of “intuition” emerge. So, different people, different organizations, etc., must be expected to have different types of intuition.

To start our analysis, consider how information is transmitted. An extreme setting has a single input that requires precision. As a single input does not admit an intuition region, “intuition” must reflect the “similarity” structure of associated events. However, if this similarity structure is missing (the precision assumption), then so is intuition. An illustrating example is where a computer requires each interpretation to hold only for a single event with zero tolerance for errors (i.e., there is no definition of proximity). Although this precision permits rapid computations, it also empties the intuition regions of Fig. 3. Thus precision without similarity kills intuition! “Intuition” can be partially re-introduced to computers through the programming of “heuristics”; these are “rules of thumb” identifying what events are similar to other events. Thus, heuristics can be viewed as attempts to recapture aspects of the Fig. 3 intuition.

For another example, consider Piaget's general and discriminatory assimilations. Assuming a “similarity” structure and postulating the existence of nodes with these properties requires one Fig. 3 oval to represent all events that are similar to an input in terms of general features, while the second oval emphasizes details. The intersection, or region of general agreement, is where both are satisfied. As the remaining regions represent types of intuition, one intuition region emphasizes events which agree in general terms while the other concerns events which agree in certain details (say, in details of a proof).

Extending this analysis to organizations is easy. Here the intuition regions may correspond to similarities in how a product is produced, or how it is marketed, or how it is financed, or . . . depending on the different divisions of labor and decisions.

If a person is intuitive in one area, will that person exhibit intuition in another? According to this structure, the answer is “Not necessarily”. Our definition of intuition heavily depends on the “association” regions, so it is based on the particular input. The way a function is defined on a certain region of a space need not dictate how it is defined elsewhere, so the intuition regions can differ with different inputs.
Assisting intuition

This working definition for intuition suggests how to enhance “intuition” and, hence, “creative thinking”. The idea is obvious; adopt procedures which encourage the exploration of each intuition region. While the exact manner this is done depends on the structure of $BB$ and the input nodes, the general idea is to “turn off” the evaluation. Remember, the intuition regions are where not all critical comparisons have been made. So, momentarily turning off certain input nodes while considering implications of others allows “fuzzier” comparisons; new associations are included even though they may be contradicted by other information (i.e., comparisons with other nodes). Sound familiar?

How this is done depends on the $BB$ structure. To see this, consider an organization where separate units report to a central decision unit. This division of responsibility allows charging the different divisions (the different ovals) to separately consider options without worrying whether they are compatible with options from other divisions. What makes it easy to describe intuition searches for organizations is that evaluation stage can be turned off and on. The same is true for any $BB$, even individuals, allowing such a decentralization. This “turning off” approach is captured by the common refrain heard in a mathematical research session of “For the moment, forget about the fact that...”

Intuition searches become more difficult with $BB$ choices where the evaluation is not separate. In particular, for humans, rather than being conscious acts, many decisions are made sub- or unconsciously. (In “Adaptation”, this subconscious activity is modeled by adaptation.) When this happens, a conscious act is needed to partially suspend judgment until after the various options, or “intuition regions” are explored. Notice how this description corresponds to the approaches mentioned in the introductory paragraph of this essay. Indeed,

brainstorming, which requires individuals to suggest resolutions without making judgments as to their feasibility or effectiveness, clearly is a conscious manner of ignoring evaluation until after the intuition regions are explored.

Similarly, the purpose of

abstraction, with its emphasis on discarding all traits that are not essential, is to enlarge the similarity Organizational regions associated with each $e$. Again, this creates larger intuition regions.

A closely related approach widely used in mathematical research is

simplification, where the actual problem is replaced with a much simpler version. By removing the complexity of the original problem, more connections can be made (so, the “proximity” regions are increased in size)

and judgement about how to resolve the original issue is delayed (information about the original problem is ignored).

Other methods admit similar descriptions. For instance,

different framings, by placing an object in a different framework, change the associated emphasis of the Organization; it requires new environmental events to be in the oval regions.

Also notice how this approach supports ideas from education (e.g., trying to create the appropriate atmosphere in the classroom where risk-taking becomes socially acceptable) where the emphasis is on exploring options.

ADAPTATION

We now include adaptation. We need this notion to reflect the realism that an input $e \in E$ cannot be instantly assigned an interpretation $b \in BB$; instead $b$ should result from a search. Of the many possible models, including trees, we examine “adaptation”. Namely, we describe searches that start by “adjusting” the current situation. This assumption leads to several natural mathematical structures; a partial selection and their consequences follows.

Think of adaptation in terms of a metal ball attracted by magnets. After firmly attaching several magnets, suppose a metal ball is placed on the smooth table. This ball may be attracted toward one of the magnets, but which one, and how long it takes depends on the attracting strength of each magnet and the initial position of the ball. A more interesting setting is if we had a variety of magnets where each type attracts only certain kinds of metals, but not others. This more general setting captures the sense of our modeling.

Mimicking the metal ball, adaptation requires an initial perception of an event to be refined and attracted toward the best fit of an interpretation. How long this adjustment takes depends upon the capabilities (the wiring) of $BB$, the input nodes, and the interpretations (magnets). Complicating the issue is that each input node adjusts the interpretation. Thus, simultaneously the interpretation of each node may be pulled in different directions (i.e., toward different final interpretations). This structure allows all sorts of scenarios. For instance, one could imagine a setting where one node is near a final interpretation when the effect of another node suddenly and radically changes the interpretation. Indeed, one can even construct scenarios with alternating interpretations!

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13 Adaptation is modelled as a dynamical system determined by the structure of its attractors.

14 This describes a standard hyperbolic structure where the motion moves near the stable manifold until, near the equilibrium, the unstable manifold begins to dominate.
Admittedly, this scenario seems preposterous. To show that it is not while making these comments a tad more concrete, stare for awhile at the Necker cube of Fig. 4. The initial well-defined interpretation allows you to specify whether a particular corner (say, the vertex about the “N”) is in the back or front. But, after staring long enough, the orientation suddenly reverses; the corner differs from what it was! And, the orientation changes again, and again, and again. Because sight uses several nodes to transmit information, this “spontaneous restructuring” phenomenon illustrates our cyclic scenario caused by an adaptation conflict in “final interpretations”. (Each node provides a “local interpretation” of an image; the full picture requires coordinating the pieces. As the top and the bottom skewed rectangles could be interpreted as hidden or observed, the flipping occurs. By exploiting this “local interpretation” for each node, other illusions are found by creating a picture separating conflicting “local” information; Fig. 5 is another famous one.)

A more standard scenario is where a final interpretation is quickly reached. Here, the sense of “adjustment from the current situation” introduces efficiency for routine searches. (“Routine” is where what happens next is closely related (for BB) to what is happening now.) For such inputs, rather than engaging in a lengthy tree search, adaptation saves time by adjusting a current interpretation. Just as the initial position of the metal ball affects the final choice of an attracting magnet, this adaptation modeling introduces a near-neighbor prejudice for how BB interprets events. (To appreciate the BB dependency on near-by interpretations, let BB model a person’s first visit to a radically different culture. Rather than quickly handling and interpreting routine events, the novelty can cause disruption.)

An immediate “creativity” implication comes from this “near neighbor” adaptation property where the interpretation of an input can vary depending on the situation. If BB is interpreting the environment when a new input e occurs, then e’s initial interpretation is influenced by the current status and focus of Organization. With a BB structure rich enough for e to have multiple interpretations, the final belief can critically depend on the current setting.

(Thus corresponds to how the current location of the metal ball determines which magnet is the final attractor.) This adjustment property, then, suggests that a final interpretation can be influenced even by seemingly superfluous background “noise”. On the positive side, this captures the “triggering” phenomenon from creativity. On the negative side, a desired interpretation could be disrupted when pulled away by “noise” or distractions; e.g., recall the frustration of losing an idea by politely listening to someone else first. This dynamical argument captures the flavor of Mozart’s (Hadamard, 1945, p. 16) description of a creative moment. “Then my soul is on fire with inspiration, if however nothing occurs to distract my attention”. Also, this “local prejudice” suggests explanations for the “fixation” obstacle of creativity.

How quickly can an interpretation be attained? While it might be almost instantaneous with computers, the adjustment speeds of different units (i.e., different input nodes) of an organization can vary depending upon the task and the information and decision structures. With individuals, differences occur with different people and different inputs; e.g., the speed of interpretation for sight and sound differ. Thus the modeling of speed requires specific assumptions about BB, the choice of e, and the way information is conveyed to BB (the input nodes).

We take a different approach by appealing to dynamical systems for guidance about the general dynamics. Here, it is safe to assume that rate of convergence to an interpretation increases the closer it is to a non-conflicting “fit”. Conversely, if an initial perception is “far” from a final, non-conflicting interpretation, the weaker attraction and slower rates of change make the final interpretation more susceptible to interpretations from other nodes and other influences.\textsuperscript{12} (With the magnet analogy, these assumptions state...)

\textsuperscript{12} Included in this instability statement is where slight changes in position force an initial interpretation into a different basin of attraction.
that the closer the ball is to an attracting magnet, the faster it moves. Conversely, a ball that is far away moves slowly. With several competing magnets, such as when the ball is in the center of a ring of magnets, a slight change in its position can radically change the identity of the attracting magnet. Identifying "magnet" with final interpretation and the ball with an "event" provides a sense of the assumptions while suggesting conclusions.)

Reaching interpretations

Adaptation occurs in BB, so each adjusted BB position represents a momentary interpretation of e. Remember, each node pulls this short-lived interpretation toward its preferred attractor. Thus, similar to a flip of the Necker cube, a momentary interpretation acceptable for one input node but not another, can be changed.

We now can resolve the mystery of how a final interpretation is reached. An interpretation is dragged out of an intuitive region by the adaptive force of a node that finds it conflicting. This continual refinement of the transitory interpretations finally settles at an interpretation reasonably acceptable for each means of transmitting information. (The final resting position could be a compromise point; e.g., a metal ball placed halfway between two magnets remains there. Elsewhere we show how these positions cause new interpretations.) Namely, the adaptation dynamics create the aggregated interpretation of Fig. 2! On the other hand, adaptation drags momentary interpretations from the "intuition regions", so it is adaptation that must be partially countered for "creative actions". (This, we believe, is where "personality traits" may play a major role.)

As an application, recall the common creativity trait, related to the "aha" experience, where first the general outline, but not the details, of an idea are discovered. This is described in Mozart's letter where "[o]nce I have my theme, another melody comes, linking itself to the first one, in accordance with the needs of the composition as a whole ... The works grows; I keep expanding it, conceiving it more and more clearly ... It does not come to me successively, with its various parts worked out in detail ..." A similar emphasis on the general theme is when Poincaré, describing an important mathematical discovery, states "I did not verify the idea; I should never have had time ... On my return to Caen, ... I verified the result at my leisure". With so many other examples, it is reasonable to accept that the general theme precedes the details. Why?

One explanation emphasizes the competition of input nodes; each way of transmitting information excludes certain interpretations during the adjustment process. Thus, different adaptation speeds, or the tendency to rest at one interpretation before moving to another (as manifested by Necker's cube), generate dynamical behavior resembling these descriptions. Alternatively, if one applies our adaptation assumptions to Piaget's notions of generalizing and discriminatory assimilation, where the former moves more rapidly than the latter, then the dynamics requires the general picture and theme to be followed by the details.

Behavior during adaptation

Before exploring other adaptation consequences, we need to interpret the changing meanings in terms of the rate of adjustment. With a rapid adaptation, these fleeting interpretations are quickly replaced by the final interpretation. Here, the momentary interpretations are not much of a factor; an individual or organization may not even notice them. A slower adjustment, however, requires lingering at each transient interpretation. As these interpretations represent disequilibrium and conflict, they may be manifested by a sense of confusion where "confusion" depends on the characteristics of BB.

How long this "temporary confusion" lasts depends on how alien e is from the current focus. (With magnets, this corresponds to their strength and the position of the ball.) If the new e is close to the current BB focus (where "close" is in terms of the BB structure), then the adaptation assumption prejudicing the initial search in favor of a current focus requires a quick interpretation. Again, this captures the ease of interpreting daily events. Conversely, the more alien e is from the current focus of BB, and the slower the adaptation process, the longer the confusion time. An example illustrating this slower effect where BE is an organization is when you made an unusual request for an airline ticket, or a nonstandard order at a restaurant. Events within normal operating procedures are easily dispatched. But, depending on the efficiency (i.e., the adaptation speed of the organization decision and informational structures), an event out of the ordinary can create difficulties.

For individuals, inputs fitting within a common structure are quickly assimilated; those that differ from the current focus take longer to comprehend. A common example is when you quickly and unexpectedly encounter an acquaintance in an unusual surroundings—say, your neighbor from Chicago walking out of a Stockholm restaurant. Here the sense of confusion can even be manifested by momentary dizziness. Another example comes from the belief that certain LD difficulties reflect a slower adaptation rate. Support for this hypothesis comes from an effective teaching technique that is recommended for classrooms with both "regular" and "LD" students. Rather than the usual approach of calling on a student and asking a question, the order is reversed and a time gap is introduced. Namely, after a question is asked, and a short time span (allowing for adaptation), a student is called upon. Our adaptation modeling supports the observed positive reaction.
Environmental change

If environmental inputs can change, we must expect the relative difference in environmental and adaptation speeds to play a critical role in interpretations and creativity. If environmental events change more rapidly than the adaptation process, then the interpretations have to blend together; different events are interpreted as being connected or essentially the same whether or not they are. For instance, a movie projector projects a series of still pictures faster than our adaptation speed, so the still pictures become connected into what we perceive as a continuous motion. But if the projection speed is slowed, the continuous illusion is replaced by individual frames; events intended to be connected are not. The faster adaptation rate, then, forces this natural clumping to disappear—events need not be interpreted as being related. At the other extreme, events moving too fast for the adaptation rate, such as a movie in “fast forward”, cause a blur and confusion—as too many events are being connected, general interpretations are unavailable. Examples illustrating both kinds of behavior come from the LD literature.

These comments suggest that portions of creativity result from having the correct levels of adaptation. If adjustments are too slow, everything is a blur and relationships are missed. If they are too fast, relationships are separated.

Impressions

To test this modeling, suppose an event $e$ is “observed” throughout the adaptation. Here, we would expect the final interpretation to be as accurate as capable with $BB$. This makes sense; $e$ remains available for comparisons. Call this final outcome the stationary interpretation of $e$. Now suppose $e$ is removed before the adaptation is complete. As $e$ is not available for comparisons, the resulting interpretation is based upon the initial, partial interpretation. Call this interpretation the impression of $e$.

To compare how the impression and stationary interpretation can differ depending upon the circumstances, suppose an input $e$ is highly alien to the current $BB$ focus. Our representation of adaptation requires the initial search for an interpretation to be based on the current focus. So, if the input is quickly withdrawn, the mathematics requires a wide gap between its “impression” and its “stationary interpretation”. Moreover, by not keeping $e$ in the region of current attention, the initially slower search should be manifested by confusion.

There are many supporting examples resembling this theoretical prediction; e.g., it is not uncommon after an accident for a person to be unable to recall details. Or, as described earlier, recall that short period of confusion

when you unexpectedly encounter an acquaintance in a different country. A more interesting example is the widely discussed “eye-witness” phenomenon. Typically, a test group is quickly shown a picture of, say, a subway scene depicting a well dressed black business man being threatened with a knife by a ill-dressed, unsavory white man. With surprising regularity, when the picture is shown only momentarily, the race of the victim and assailant are interchanged—something that does not occur should the picture be shown for a longer period. This behavior, of course, is consistent with the description of “impression”. Personal prejudices, perhaps personal experiences, determine the “nearest magnet” for this input. When it is not there for consultation and comparison, the wrong interpretation is reached.

Related examples come from trying to identify a person from pictures. The dynamics suggest that the observed pictures should distort the memory of the actual person; forensic artists are beginning to understand this fact. This, too, is consistent with prejudice of adaptation to the current Organizational focus.

Thinking is not thought

It is this connection between adaptation and its dynamical prejudice toward the status quo that provides insight into some of those “thinking is not thought” types of behavior. Because “thinking” involves abstraction, treat the space of conscious thought as the environment $E$ where the inputs are individual thoughts. Thus, for this discussion, “thinking” is a conscious change of the $E$ inputs. The rate of change of these inputs corresponds to “environmental change”. Accompanying these change is the adaptation process which tries to eliminate conflict by reaching a final interpretation.

Our comparison of “thinking and thought” involves different, potentially conflicting dynamics. Conscious thinking, is the act of changing or holding fixed a particular $e$ (that is, changing or fixing an idea). The second, a sub- or preconscious activity, is the adaptation process which tries to reach a nonconflicting interpretation. Predictions about resulting behavior comes from our analysis of the interaction between environmental and adaptation rates of change. The adaptation prejudice favoring the status quo allows events (thoughts) which are minor modifications of current interpretations to enjoy a smoothly working procedure. Here, the manipulated inputs (“thinking”) lead to reasonably quick, smooth interpretations. But if an input (the process of thinking) is far from an accepted interpretation, the adaptation process (thought) is slow. Here the environmental changes dominate the adjustment (adaptation). Therefore, rather than reaching an interpretation, “thinking” actually hinders the adjustment process from reaching a final interpretation. This is true whether the consciously manipulated inputs are rapidly changing, or held fixed. But if environmental changes (thinking) are placed on hold—because the person is taking a walk, sleeping, attending a
play or concert — the conflict between environmental change and adaptation is removed. This allows adaptation to search for an interpretation. By the properties of the dynamic, the closer adaptation comes to an interpretation, the faster the convergence. The dynamic, then, should be manifested by a person suddenly and unexpectedly arriving at a conclusion! In other words, this adaptation dynamic captures, in a natural fashion, the “aha!” experience.

(Using the magnet description, identify the manipulation of conscious thought with the conscious movement of a loosely held metal ball. If the moving ball approaches a magnet, it will be pulled away by an interpretation — a magnet. However, if the ball is moved far from any magnet, it is doomed to remain in a confusion zone. This particular dynamic corresponds to fixing on a particular topic — the loosely held ball is kept in a region far from any magnet. Now, suppose even in this region of weak attraction, the hand is removed. If there is an attracting magnet, even if the attraction is weak, the ball will move in particular direction. Without interference, the moving ball eventually reaches a stronger attraction zone where it suddenly moves rapidly to a magnet. Aha!)

This “aha” prediction follows directly from the interaction between the two types of dynamics. This is not mystical; all of us have experienced it. All of us have suddenly, when not trying, finally remembered the name of a person, or movie, or address that earlier we could not recall. This same trait is accorded a higher status when it produces a “creative product”. For instance, compare this theoretical construct with Poincaré’s comments that “the change of travel made me forget my mathematical work . . . At the moment when I put my foot on the step (the important mathematical) idea came to me, without anything in my former thoughts seeming to have paved the way for it”. (In Poincaré’s speech reported in reference (Poincaré, 1952).) This is the “aha!” experience.

So, behavior surprisingly reminiscent of the “aha” behavior is a natural prediction of the conflicting dynamics of consciously manipulated thoughts and adaptation. Moreover, this explanation remains mysticism. Replacing the popular “light bulb” where a powerful idea suddenly occurs out of nowhere, the dynamics requires the basis to exist. For instance, this description does not permit either of us coauthors to wake with a grand, new idea in chemistry. Instead, it requires the basis for a new idea to already be in BB the ideas cannot occur without preparation.

An advantage of this model is that it suggests ways to enhance this “aha” creativity experience. To introduce an “adaptation period”, take a walk, go swimming, get involved in some activity where the mind is not consciously trying to manipulate the outcome of an idea, and let adaptation do its job. Because adaptation is highly dependent upon an initial position, this dynamic also suggests doing a little work or thought about an aspect of a problem just prior to an adaptation period; that is, just prior to going to a play, concert, to sleep, on a walk or drive.

TOWARD A MATHEMATICAL MODELING OF CREATIVITY

This strong dependency on the choice of e suggests still other strategies. Returning to the story of the metal ball, it sure would help if the ball is moved into a region where attraction is more stronger and more likely. From our earlier discussion, this suggests exploring the different intuition regions to increase the likelihood of success of the adaptation dynamic. A natural strategy is to reformulate a problem. By definition, a reformulation introduces other associations in Fig. 3. Or, rather than trying to find new associations with a problem, find associations with an associated problem. For instance, by simplifying a problem, the actual problem is related to a much simpler model. This simpler model may allow new and/or extended intuition regions.

SUMMARY

The many seemingly different and sophisticated models for cognition, creative activities, and organizational behavior have certain traits. Somehow and in some manner, information from the outside has to be conveyed and it cannot be instantaneous. By analyzing these assumptions, we discover behavior that is surprisingly similar to many of the mysteries of cognitive processes and creativity. By using standard assumptions, these consequences must be subtle, hidden properties of other models!

This particular essay provides a flavor of this type of analysis. Rather than a complete story, only particular aspects of a more general model are described with suggestive examples and description.

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