Introduction

Let me begin by stating why I became an active participant in this discussion of the foundations of decision analysis. For many years, I had been blithely practicing in the field in the belief that all important questions about the underpinnings of the subject were now thoroughly understood and generally agreed upon. I knew that various theoreticians were developing alternate axiomatizations of decision theory, as they always had, but I did not believe that the resulting theories were being seriously proposed as bases for decision making. The onset of foreboding occurred when a prominent theorist made clear to me that he would reject a commonly accepted norm of decision analysis in making certain of his own decisions. I have previously described the conversation with the theorist that initiated my current militancy (Howard, 1988). However, I shall repeat it now for easy reference and because it so succinctly illustrates the issues at hand.

The choice situation offers two options, A and B, for receiving monetary payments depending on the role of a die. The die has six faces numbered 1 through 6, and you have no basis on which to assign other
than equal probability to the six faces. The possible monetary amounts you can receive are $500, $600, $700, $800, $900, and $1,000, according to the face that is tossed. However, the monetary payment associated with each face depends on the option you play as shown in Table 1.

If you play with option A, then face 1 pays $600, face 2, $700, up to face 5 which pays $1,000 but then face 6 pays $500. If you play with option B then face 1 pays $500, face 2, $600, etc., up to face 6 which pays $1,000. The question is, given a choice, which option do you prefer. The theorist said that for the problem as stated he would prefer A over B because for five out of the six possible faces he would win more with A than B. However, if the payoff were in thousands of dollars rather than in dollars, then he would prefer B over A, because of the great regret he would experience should the die produce face 6.

When I recounted this conversation to my decision analytic colleagues, they thought they must have misunderstood what I was saying because they could see no difference between the two options. They simply said that each possible payment had a probability of one-sixth regardless of the option, so what difference would it make which option was selected. If the theorist had been any less distinguished, or if he had suggested that his choices were descriptive of certain people’s behavior but not what he would do when playing for real dollars, then there would have been no reason to be concerned about the discrepancy between his opinion and my own. But under the circumstances, the difference of opinion was troubling. Upon further inquiry, I found that the theorist was not alone in proposing that people should depart from the norms that have for so long guided decision analysis. This finding required me to examine my position and either to change or to affirm my views. The examination has led me to affirm my belief in the fundamental rules of decision analysis and to resolve to argue cogently for them in an attempt to correct what I perceive as an unproductive heresy.

To this end, I shall first discuss some of the basic issues, terms, and concepts that must support any discussion of the rules for decision making. Then, I will present the rules that we currently use in a simplified
form. Next, I will discuss the desiderata, both essential and desirable, for any formally defined decision-making system, and show the extent to which our current rules satisfy the desiderata. I shall then describe in general terms how other proposed rules fare when considered against the desiderata. In particular, I shall reconsider two well-known problems posed by Allais (1953) and Ellsberg (1961) that are said by some to require new rules for their proper treatment. Though these problems are sometimes called paradoxes I shall refer to them as phenomena, since in my view they reflect a need for education rather than a flaw in the theory. I shall show that these phenomena require only a careful application of our rules, and not a revision of them, and that rejection of the rules would led to a violation of some of the most fundamental desiderata for decision making. I end with a discussion of whether the rules apply when “equity” is a consideration and comment on their use in group decision making.

Background Considerations

Descriptive versus Normative Theories

A fundamental distinction we must agree on is that between descriptive and normative theories. A descriptive theory is one that purports to describe the world as it is. Its quality is measured by the extent to which it accurately characterizes and predicts the behavior of actual systems, and in the present context, the behavior of people. Thus, the inverse square model for the gravitational attraction of two bodies is an excellent descriptive model for the behavior of many planetary objects, although it may be difficult to apply to some problems, for example, the classical three-body problem.

A normative theory, on the other hand, establishes norms for how things should be. Therefore normative models have no place in the physical sciences because they deal with fact rather than with human will. However, for decision-making situations, where human will is very much an issue, then it is not only possible but often desirable to propose norms or standards for decision making so cogent that, once accepted, any departure from them will be regarded as a mistake.

We should note that normative theories are important primarily when our natural behavior is not satisfactory to us; that is, the behavior does not satisfy the norms that we have for the process in question. To illustrate, suppose that we developed a normative theory of breathing.
The theory would tell us to breathe more rapidly when we run upstairs and
to breathe more slowly when we are lying at rest. But since this is the
behavior that we would naturally exhibit, few of us other than specially
trained athletes and singers are likely to enroll in a course on breathing.

The need for normative aid seems to arise particularly in questions of
uncertainty and decision making. In the area of uncertainty the roll of
people who have made mistakes in probabilistic judgments reads like
the roll of the geniuses of science. It appears that most humans are
condemned to second-rate thinking where uncertainty is concerned. For
example, the relationship between cigarette smoking and lung cancer is
often seen as an associative logic linkage: If you smoke you are likely to
get lung cancer; if you get lung cancer you are likely to have been a
smoker. The conditional probabilities of cigarette smoking given lung
cancer and of lung cancer given cigarette smoking are actually quite
different, but without the precise formulation afforded by the notion of
conditional probability it is difficult even to have the discussion. Upon
seeing an analysis showing that the chance of getting lung cancer, given
that you are a smoker, is considerably less than the chance of having been
a smoker, given that you get lung cancer, a postdoctoral M.D. remarked
that these results were impossible because his lung cancer ward is full of
cigarette smokers.

As many cognitive psychologists have observed, the situation does not
improve when we consider performance in decision making, which
involves not only uncertainty but issues of preferences. Classroom study
in decision analysis is heavily involved in purging such cognitive errors as
including sunk costs or failing to recognize dominance. The reason we call
them cognitive errors is that they violate the normative paradigm.

We should not be surprised that education is required to align our
thoughts with what is so. When I first encountered in high school science
classes the notion that the pressure of a body of water depended only on
its depth and not on the size of the body, I could not reconcile it with
common sense. I said, “You mean that Boulder Dam would have to be
just as thick even if Lake Mead extended only one foot behind it rather
than for dozens of square miles?” I still remember walking home from
class and thinking of a thin wax paper barrier situated exactly one foot
behind Boulder Dam and realizing that the pressure would be the same
on both sides, so that the existence of the rest of Lake Mead really did
not make any difference. Then it became obvious that pressure depended
only on depth whereas that morning it had been “obvious” that it did not.

Since we appear to be condemned to using instruments in matters of
uncertainty, just as is a pilot when flying in bad weather, we should be
grateful that we have had such an excellent set of instrument builders. In the area of probability I am grateful to many but particularly to Laplace, Jeffreys, and Jaynes for providing what I consider to be the most useful underpinnings of the study of uncertainty. In the area of preference, particularly for preference where risk is concerned, I am indebted to Daniel Bernoulli for the form of the result and to Von Neumann and Morgenstern for the mathematical justification of the form.

The theory of decision based on these foundations is normative: the theory says how you should act if you wish certain properties to be true of your behavior. The theory can be used as a descriptive theory, and then the test is whether it describes what people do. Decision analysis is the professional discipline based on the normative decision theory. A decision analysis might use a descriptive model of other decision makers if that model adequately characterized their behavior (not too likely, my psychologist friends tell me). There is no need for this descriptive model to have any relation to the normative theory on which decision analysis is based.

Characterization

To describe decision making we must first characterize the decision situation. We begin with the notion of an epoch, a point in time at which the thought about the decision is taking place. We characterize the future by creating different kinds of distinctions (Spencer-Brown, 1979) with two or more degrees of each kind. To determine whether a distinction is sufficiently defined we use the clarity test (Howard, 1988). A distinction meets the clarity test if a clairvoyant with the power to know any physically definable event or number in the future or the past could say which degree of the distinction occurred without any use of judgment.

With these kinds and degrees of distinction we create possibilities: the mutually exclusive and collectively exhaustive collections formed by choosing one degree of each kind of distinction. To each possibility that we have created we may associate one or more measures (numbers of interest that are connected with this possibility). We assess our degree of certainty on each possibility by dividing our certainty among successive distinctions, a process we call assigning a probability to each possibility.

Thus for each course of action or alternative we have a description of potential futures in terms of possibilities and their associated probabilities. Each possibility that we have created we think of as a prospect, that is, as a future life whose salient features are described by the
possibility we have defined, but which is, in fact, imbued with many uncertainties and future choices. I use the word prospect rather than possibility when discussing preferences over futures rather than just their logical existence. “Prospect” connotes the “looking forward” to a future about which we have preferences. Note, too, that a prospect can include effects on other people, the environment, etc.

For each alternative we shall use the word deal to describe the range of prospects and their associated probabilities. Thus the essence of decision making in this characterization is to choose among alternatives, each described as a deal.

Choice

The question is what process to use for choice among deals. We seek decision quality, that is, quality thought about the decision that will lead to clarity of action. We must realize that the map is not the territory, that the abstraction we have made of the decision process is not an accurate and complete representation of the decision situation, but rather a means of achieving clarity of action.

Our task is to select rules for making the decision such that if these rules are followed we will be sure that no process errors have occurred. We can think of this as selecting rules for an agent who will make the decision on our behalf. We would clearly not want to indulge the idiosyncrasies of the agent by allowing him to use rules of which we did not approve.

A proper set of rules, or decision theory, is indispensable to decision analysis; however, it is only a part of decision analysis. The professional skills of the analyst are required to be sure that the decision is properly framed, or better, to frame the best decision (Matheson, 1990). Decisions do not occur naturally—they are acts of will: decision opportunities, or, more briefly, decisions are declared into existence by the decision maker. Helping the decision maker declare the right decision at the right time is the essence of effective aid in framing.

The analyst must also be skilled in eliciting creative alternatives, accurately eliciting and assessing information, and in representing appropriately the preferences of the decision maker. The result will be the formal description of the decision problem, the decision basis. However, our current concern is with the normative rules to be used in making the decision.
The Rules of Thought

I shall now discuss the rules of thought that I consider to constitute the foundation of decision analysis. These rules apply to thought about making a decision at a particular epoch; to the extent that future decisions are involved, the thought about them is thought that occurs at the present epoch.

The Probability Rule

The first rule is the probability rule. This rule states that deals will be described by possibilities and probabilities, or equivalently by prospects and associated probabilities, as outlined above.

The Order Rule

The second rule is the order rule, which states that all prospects of all deals under consideration can be ordered in terms of preference. We can visualize this as placing them in a list with the most preferred prospects at the top and the least preferred at the bottom. It is permissible for two prospects to be at the same level in the list, that is, to consider oneself indifferent between them. What is not allowed is that the same prospect appear at two different levels in the list.

The decision basis need not remain unchanged in this process. We can consider this rule as a commitment to introduce sufficient distinctions into the basis until the resulting prospects can be ordered according to the rule’s requirements.

The Equivalence Rule

The third rule is the equivalence rule: If three prospects $A$, $B$, and $C$ are at different positions in the list with $A$ above $B$ above $C$, then you can specify some probability that will make you indifferent between receiving prospect $B$ for sure and receiving a deal that has probability $p$ of yielding prospect $A$ and one minus $p$ of yielding prospect $C$. The probability $p$ is called a preference probability because it does not correspond to the uncertainty of any event in the world that could be revealed by a
clairvoyant. The prospect $B$ is said to be the certain equivalent of the deal with probability $p$ of $A$ and $1-p$ of $C$.

Satisfying the equivalence rule may once again require introducing new distinctions into the basis. For example, I may be clear that I prefer a two week trip to Tahiti to receiving $100 to receiving nothing. However, when I am asked what chance of a two week trip to Tahiti versus nothing is just equivalent to $100, then I may have to inquire more about the trip. I may want to know about accommodations, meals, tours, etc. and thus create new distinctions and consequently new prospects before I am ready to assess preference probabilities.

**The Substitution Rule**

The fourth rule, the substitution rule, states that you are indifferent between receiving a prospect for sure and receiving any deal for which it is the certain equivalent. What this means is that you may use preference probabilities in all calculations as if they were probabilities of real events.

**The Choice Rule**

The fifth and last rule is the choice rule, the only rule that requires action. The rule states that if prospect $A$ is above prospect $B$ in your ordered list, and if you face two deals with these same two prospects, but the first deal gives you a higher probability of prospect $A$, then you must prefer the first deal to the second. In other words, you must prefer the deal with the higher probability of the prospect you consider more desirable.

Based on these rules and the general rules of logic, you can build an entire edifice of consequences that represent rational thought about decision making.

**Axioms and Theorems: The Decision Composite**

Let us stop to examine this process of construction. In developing a formal theory, certain properties are postulated as axioms, ideas so simple and transparent that there is no question about whether you would wish to follow these ideas in situations that were entirely described by them. The rules of thought we have discussed are the axioms of
decision theory that we shall use. They correspond in the development of Euclidian geometry to such axioms as the idea that a straight line is the shortest distance between two points. Based on the axioms, we develop, in decision theory and geometry, a number of theorems that must be true, respectively, for decisions or figures in a plane; the axioms imply the theorems. We often have a choice in the development of a subject as to which set of axioms we should use, a choice usually based on esthetics and on the ease of proving the theorems from the axioms. If some theorems are taken as axioms, then some axioms may end up being theorems. We must look at the entire collection of axioms and theorems to judge the usefulness of the formal structure.

In the case of decision theory, I shall call this collection of axioms and their implied theorems the decision composite. Since the choice of what is an axiom and what is a theorem is, as we have said, somewhat arbitrary, we should really be concerned with the desirability of the decision composite. We shall now turn to this question.

Desiderata

I shall list the properties that I consider desirable for any decision composite. I shall proceed by first discussing the properties that are essential in any situation, then those that are essential in particular situations, and finally those that are, if not essential, then of great practical importance.

Essential Properties. The essential properties proposed for the decision composite are:

1. The decision composite must allow me to form inferences about uncertain distinctions even in the absence of a decision problem. This means that probability must stand on its own epistemologically.
2. The decision composite must be applicable to any decision I face regardless of type or field. Once a student remarked, “I can see that these rules would be useful for investments, but surely you are not proposing them for making important decisions about the health of your family.” I replied, “These rules are the best way I know to make decisions. My family deserves the best. In matters of life and death, I do not want to make a mistake.”
3. The decision composite must require that I prefer a deal with a higher probability of the better prospect, in conformity with the choice rule we have discussed. Note that even a masochist will desire to follow the choice rule in the pursuit of abuse and physical pain.
4. The decision composite must require that I be indifferent between two deals that I consider to have the same probabilities of the same prospects. In other words, I should be indifferent between two deals for which I have created the same descriptions.

5. Reversing the order of contemplating uncertain distinctions should not change inference or decision. This means, in particular, that changing the order of receiving a given body of information should not change any inference or decision. For example, if I received a set of data A and thereafter a set of data B, then I should assign the same probability on future distinctions and make the same decisions as if I had received first set of data B and then set of data A. This property is sometimes called Invariance to Data Order. It is so appealing that it might be taken as an axiom, a rule of thought.

6. If I prefer alternative (deal) I over alternative II when the uncertain event A occurs and if I prefer I over II when A does not occur, then I must prefer alternative I over II when I am uncertain about the occurrence of A. Furthermore, I want this to be true even if I am indifferent between I and II for one of the two possibilities for event A. To illustrate, if I would prefer to drive my car to work rather than to ride my motorcycle when it rains and if I am indifferent between the two means of transport when it is fair then if there is any uncertainty about the weather my decision should be to drive the car.

7. Once I have selected my best alternative, the non-informative removal of the opportunity to choose any other alternative must not affect my decision. Non-informative means that the change does not provide information about the other dimensions. I sometimes illustrate this property by considering the case of someone who has entered an ice cream parlor and inquired about the flavors for sale. The server replies that vanilla, chocolate, and strawberry are available. The customer says, “I’ll have chocolate.” A moment later the server returns to announce, “I am sorry, we have no strawberry.” The customer replies, “In that case I’ll have vanilla.” Such behavior would not be consistent with this property, which is sometimes called the Independence of Immaterial Alternatives property.

8. The addition of a new alternative to the basis cannot make an existing alternative less desirable to me. That is, while a new alternative may well improve the decision situation, it certainly cannot hurt it. In the extreme, I can simply ignore the new alternative and chose the one I previously considered best.
9. The possibility of obtaining clairvoyance on any uncertainty cannot make the decision situation less attractive. The emphasis here is on decision situation. I may well be engaged in doing a puzzle or waiting in line to see a mystery movie and decline to be told either the answer to the puzzle or the villain in the mystery without violating this property. In other words, in any situation in which suspense is not a part of the enjoyment and in which I face a choice, I should welcome new information, for it can never hurt and may well result in my obtaining a better deal.

10. At this epoch my thoughts about how I will behave and choose in the future must be consistent. For example, if I believe that I will prefer $10 over $5 tomorrow and I am making a decision today that may lead me to that choice, then I cannot say that I am uncertain about what I will do when I face the decision. This does not mean that I will not be free to make any choice I like tomorrow or to change my tastes, but only that my present thoughts about what I will do must be consistent. This property is sometimes called Sequential Consistency.

11. It should make no difference to my current decision when I face a series of actions whether I consider the sequential process of information revelation and what I will do in the face of each such revelation, or whether I consider all possible future actions I might take and all future possibilities I might learn, with their associated probabilities, and then find the best set of actions. This property is called Equivalence of Extensive Form and Normal Form Analysis. It is closely related to property 5 discussed above.

**Essential Properties With a Value Measure.** While not strictly necessary, most decision analysis problems are formulated in a way that allows a value measure to be placed on each prospect. This value measure is a fungible and alienable resource, usually money, in which the difference between two prospects can be interpreted as the willingness to pay to change from one prospect to the other.

A resource is fungible if you are indifferent to substituting one unit or fraction of a unit for another such quantity in any deal. In other words, all units of a fungible resource are indistinguishable and divisible. Furthermore, the resource must be such that more is always preferred to less, so that higher prospects in the preference list will correspond to more of the resource. In practice, it is difficult to think of a resource other than money that meets these requirements, although in some settings measures like “time saved” may be adequate.
The value measure must be alienable, capable of being transferred to the ownership of another, if the value of clairvoyance results are to be interpreted as guidance for selecting information-gathering processes that must be contracted for. For example, you cannot pay a laboratory to conduct a test by signing over some of your "time saved."

The introduction of a value measure is useful for two reasons. First, because it allows the preference comparison of prospects to be conducted against a familiar scale of value. The second reason is that it allows the possibility of computing a value of clairvoyance in terms of this measure on any uncertainty or collection of uncertainties in the problem.

When we use a value measure we can add the following essential properties for the decision composite:

1. **Since the value measure is chosen so that more will always be preferred to less, the higher the prospect in the prospect list the higher must be the value measure.** This ensures that there will be no possibility of making me a "money pump" if I follow my expressed preferences.

2. **For each deal I must be able to compute the amount of the value measure I would have to receive in exchange for the deal in order to be indifferent to selling it.** This threshold selling price of the deal I shall call the certain equivalent. This property requires that certain equivalents exist.

3. **I must be able to compute the value of adding any new specified alternative to the decision basis: the value must be nonnegative.**

4. **I must be able to compute the value of clairvoyance on any uncertain distinction or collection of uncertain distinctions; the value of clairvoyance cannot be negative.**

5. **Payments of value that cannot be changed must have no effect on future decisions.** For example, except for informational and tax effects, the price for which I am just willing to sell something must not depend on what I paid for it. More generally, when I am ranking prospects that I consider to be equivalent to changes in wealth, how I arrived at my present state of wealth must not matter. This is the "sunk cost" principle.

6. **There must be no willingness-to-pay to avoid regret.** My preferences must be on prospects—the futures I face. Regret is a bad thought that arises when I think about futures I might have received instead of the future I did receive.

7. **At least simple stochastic dominance must be satisfied.** This means that if for any $x$ the probability of receiving a value measure of at least $x$ with alternative I is at least as great as the probability of receiving a
value measure of at least $x$ with alternative II, and if it is greater for some $x$, then I must prefer alternative I to alternative II.

You will notice that several of these properties are special cases or refinements of the essential properties when there is no value measure. For example, the requirement that the value of clairvoyance be nonnegative is just a precise statement of the requirement that the decision situation can never be made less desirable by the possibility of clairvoyance.

**Practical Considerations.** In addition to these essential properties, there are certain practical or methodological considerations that are highly desirable, if we are to have a discipline of decision analysis instead of only a theory of decision making. One practical consideration is that the individual evaluation of prospects must be possible. This means that in the language of the rules of thought we discussed earlier, the prospects can be placed in order individually rather than being considered in pairs or triplets, etc.; therefore, we can ask preference questions like, “Would you rather have $500$ than $100$?” rather than “Would you rather have $500$ when you might have had $1,000$, or $100$ when you might have had $50$?” Some proposed theories appear to require asking the latter type of question. A second practical consideration is that the methodology of tree rollback, or equivalently, influence diagram evaluation be available to solve the decision problem by determining the best deal. While in principle we could adopt methodologies that violated these practical considerations, the cost of decision analysis would become prohibitive for current uses.

**Rating the Present Theory**

As you might have guessed, the old time religion decision composite based on the five rules of thought we have discussed satisfies all the desiderata suggested, both conceptual and practical. Furthermore, and most importantly, I have heard of no other theory that satisfies the essential desiderata, much less the practical ones.

Why then is there a demand for new theory? The general reason is that some people think that it is reasonable to depart from the recommendations of the normative theory. They believe this because many people do depart from the recommendations of the normative theory in decision experiments and because in at least one case, that of equity, the recommendations seem conceptually inappropriate even to some thoughtful researchers. I personally believe that neither of these challenges constitutes an effective attack on the normative theory. I shall first discuss the experimental challenge and then the conceptual challenge.
The Experimental Challenge

Some researchers are uncomfortable because the normative theory is not a good descriptive theory of decision making. They would like to consider what is currently observed in some decision-making situations as “OK” rather than as mistakes in terms of the normative theory. I believe that just as for me in the case of hydrostatics, the answer lies in education rather than in changing the rules of the game. To illustrate I shall discuss two of the cases that are often offered to justify the development of a new normative theory.

The Allais Problem

Consider a version of a famous problem in decision theory, the Allais Problem. Suppose you are fortunate enough to have the chance to win a large sum of money as follows. According to the toss of a coin you will receive the opportunity for an attractive deal in either Room A or Room B. If you receive Room A, you will have to choose between two deals, Deal L1 and Deal L2. Deal L1 is easy to describe: if you choose Deal L1 you will receive a sure $1,000,000. Deal L2, however, is uncertain. With Deal L2 you have a 10 percent chance of winning $5,000,000, an 89 percent chance of winning $1,000,000, and a 1 percent chance of winning nothing. The chance mechanism is such that you assign these probabilities of winning each amount. Which do you prefer, Deal L1 or Deal L2? If you receive Room B, you will have to choose between Deal L3 and Deal L4, both of which are uncertain. Deal L3 has a 10 percent chance of paying you $5,000,000 and a 90 percent chance of paying you nothing. Deal L4 has an 11 percent chance of paying you $1,000,000 and an 89 percent chance of paying you nothing. Do you prefer Deal L3 or Deal L4?

Now let us suppose that you have to instruct an agent to make the choices for you in whichever room you receive. For example, you could instruct the agent to choose Deal L1 in Room A and Deal L3 in Room B. But other instructions are possible such as L2 and L4, or L1 and L4, or L2 and L3. You could even be indifferent between the deals offered in one room or the other or both. What instructions would you give to your agent? If it cost you $100 to have the agent follow your instructions rather than leave the choice to the agent's whim, would you pay? Empirically, a sizable fraction of any group will instruct the agent to choose L1 and L3. I find that people giving this instruction often say they would pay $100 or
more to assure that those instructions would be followed rather than, for example, the alternate instructions L2 and L4. Let us examine this behavior in more detail.

**Analysis.** Figure 2–1 shows decision trees for the possibilities arising in Room A and in Room B. The payoffs are in millions of dollars. Recall that Room A or Room B is chosen by the toss of a coin, so that a more accurate representation of the entire situation appears in figure 2–2. Your task of delegation is to inform your agent which deal to select in each room that you might receive. Let us examine the probabilities of winning each amount of money if you give each of the four instructions L1, L3; L1, L4; L2, L3; and L2, L4.

Figure 2–3 shows the consequences of the instruction L1, L3. They are a 5 percent chance of $5,000,000, a 50 percent chance of $1,000,000, and
a 45 percent chance of nothing. Figure 2–4 shows the corresponding consequences of the L1, L4 choice. We see that there is no chance of winning $5,000,000, a 55.5 percent chance of winning $1,000,000, and a 44.5 percent chance of winning nothing. Figure 2–5 shows the results of the L2, L3 choice, namely, a 10 percent chance of winning $5,000,000, a 44.5 percent chance of winning $1,000,000, and a 45.5 percent chance of winning nothing. Finally, figure 2–6 shows the results of the L2, L4 delegation. We see that there will be a 5 percent chance of winning $5,000,000, a 50 percent chance of winning $1,000,000, and a 45 percent chance of winning nothing. But we have seen these results before. They are the same probabilities of these payoffs we derived for the L1, L3 delegation in figure 2–3. This means that the L1, L3, and L2, L4 choices will lead to exactly the same probabilities of the same payoffs, and,
therefore, that anyone who paid $100 to insure that the instructions would be $L_1$, $L_3$ rather than $L_2$, $L_4$ would have wasted his money. Recall that we said that an essential desideratum for a theory of decision making was that a person be indifferent between deals with the same probabilities of the same prospects.

The Allais Problem fools people because it encourages them to be fearful about the 1 percent chance of losing their $1,000,000 in Room A, while it induces them to be greedy and ignore this risk when they are confronted by Room B. In other words, Room A promotes focussing on uncertainty, whereas Room B encourages concentrating on payoffs. The version of the problem discussed here requires people to be consistent in their tradeoff between greed and fear, namely, their risk preference. I consider the Allais Problem to be a phenomenon and not a paradox. The fact that people will make choices that they will later see to be mistaken
Tversky and Kahneman (1981) illustrates only the need for education, and not a need to change the normative theory. This is the lesson I learned in my adventures with hydrostatics.

**The Ellsberg Problem**

Another famous problem requires people to express preferences for payoffs that depend on the color of a ball drawn from an urn with uncertain composition; I present my own version. The urn is shown pictorially in figure 2–7; it contains 90 balls of which 30 are red and 60 are either blue or yellow. We know that the blue and yellow balls were selected from a large supply of both colors by a colorblind child. Again, you have the opportunity to receive an attractive deal based on the urn in one of two rooms according to the toss of a coin. In either room you will play a game exactly once, and must choose a payoff scheme for it.

If you receive Room A, then the payoff schemes are shown in figure 2–8. If you play with payoff scheme I, then if a red ball is drawn you will receive $100 and otherwise nothing. If you play with payoff scheme II, then only a blue ball will pay you $100. Which scheme do you prefer, or are you indifferent? If you receive Room B, your payoff schemes are as shown in figure 2–9. Payoff scheme III will pay you $100 for either a blue or yellow ball, otherwise nothing. Payoff scheme IV will pay you $100 for
30 Red Balls
? Blue Balls
? Yellow Balls
-------------
90 Total Balls

Figure 2–7. The composition of the urn.

<table>
<thead>
<tr>
<th>Ball Drawn</th>
<th>Payoff Scheme</th>
<th>Red</th>
<th>Blue</th>
<th>Yellow</th>
</tr>
</thead>
<tbody>
<tr>
<td>ROOM A</td>
<td>I</td>
<td>$100</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>0</td>
<td>$100</td>
<td>0</td>
</tr>
</tbody>
</table>

Figure 2–8. The payoff schemes in room A.
either a red or a yellow ball, again otherwise nothing. Which scheme do you prefer, or are you indifferent?

Now suppose once again that an agent will make your choices for you. What would your instructions be? Would they change if the potential prize was $100,000 rather than $100? Would you pay 1 percent of the prize amount to have your instructions followed rather than be subject to the whim of the agent? In my experience, many people prefer to play with payoff scheme I in Room A. In Room B, most people prefer to play with payoff scheme III. When both rooms are considered, many people in any group will make the choice I, III and be willing to pay a small sum for it.

Analysis. Recall that the room is selected by tossing a coin. The problem is so constructed that if you make the choice I, III, you will receive a 50–50 chance of winning $100, regardless of the color of ball drawn. Furthermore, you will receive a 50–50 chance of winning $100, regardless of the color of the ball drawn, if you select II, IV. In other words, you have expressed a preference between two situations that have the same probabilities of the same prizes. Again, this is a violation of a fundamental desideratum.

Of course, the choice of II, IV over I, III would be equally inconsistent, as would be having a preference in one room but being indifferent in the other room. The situation is illustrated by showing the nine possible delegations for choice in each room shown in figure 2–10. Of the nine delegations, six are eliminated because of their inconsistency: they are inconsistent with any state of information you could have about the urn. Suppose, for example, that you knew that the 60 balls of uncertain composition were mostly blue. Then you can see from the payoff tables that you would prefer payoff scheme II in Room A and payoff scheme III
in Room B. Conversely, if you knew that the balls were mostly yellow, then you would prefer schemes I and IV. If you could make neither statement about the composition of the urn, then your preference would have to be described by indifference between payoff schemes I and II, and between schemes III and IV, a situation described by the central box in the figure. The point is that only the three shaded diagonal boxes represent choices consistent with anything that you might know about the urn. We shall now show that only the choices in the central box are consistent with the information actually provided about the urn.

Let $B$ be the event that a blue ball is drawn. Let $n$ be the number of blue balls in the urn; $n$ can take on the values $0, 1, 2, \ldots, 60$. Then we can write:

\[
\{B \& \} = \sum \{B \mid n \& \} \{n \mid \& \} \\
= \sum (n/90) \{n \mid \& \} \\
= (1/90) \sum n \{n \mid \& \} \\
= (1/90) \langle n \mid \& \rangle
\]
We see that the probability that a blue ball is drawn is $1/90$ times the mean of the distribution on $n$ that we assign. Figure 2–11 shows some of the possible assignments we might make for $\{n|&\}$. The assignment marked a corresponds to equal likelihood that either all or none of the balls are blue. Assignment b says that each number of blue balls from 0 to 60 is equally likely. Assignment c would be the binomial distribution resulting if you said that each of the balls was equally likely to be blue or yellow. The assessment a might arise if you thought that the colorblind child simply dipped the urn into a large vat of balls, either blue or yellow, depending upon which he came to first. The assignment c might be appropriate if he wandered through the warehouse picking up a ball every now and then.

But what do we really know about the composition of the urn? What we know is that, if every blue ball in the urn is replaced by a yellow one and vice versa, the problem is unchanged. We call this the invariance to relabeling of blue and yellow balls, a case of exchangeability. This means that whatever distribution we assign to $n$ must be symmetric about $n = 30$. All the distributions in figure 2–11 meet this test as would many others—there is no justification in the problem for assigning any distribution to $n$ lacking this property. Of course, this requires that the mean of any such distribution must be 30 and therefore that the probability of a blue ball must be equal to $30/90$ or $1/3$. And, of course, the probability of a red ball equals the probability of a blue ball equals the probability of a yellow ball equals $1/3$—not approximately $1/3$, but exactly $1/3$. Thus we can say that in the problem as stated, there is no justification for delegating to your agent any other choice than indifference between the payoff schemes in both of the rooms.
It is important to note the qualification above, "In the problem as stated". On one occasion, I had a student, well-versed in the prevalence of different kinds of colorblindness in children, develop a model that led her to assign different probabilities to a blue and a yellow ball. The point is that there is no need to consider probabilistic "ambiguity." We need only recognize and characterize the information on which the probability assignment is based. Like the Allais Problem, the Ellsberg Problem affirms the need for education to guide choices that otherwise may later be seen as unwise.

The Conceptual Challenge: Equity

Some people feel that the rules of decision making that we have discussed are challenged by the notion of equity. To illustrate their concern, suppose that a mother is faced with giving a present either to her son John or her daughter Mary. She is indifferent between the prospect of John’s getting the present and the prospect of Mary’s getting the present. Those concerned with equity say that it is reasonable for the mother to prefer to flip a coin to determine which child gets the present rather than to give it to, say, Mary without such a coin toss. That is, she might prefer a 50/50 probabilistic mix of the prospects to either prospect. It should be clear that such a choice would violate the rules since all prospects are at the same level on the preference list.

There are several points that could be made in terms of this example. It is admittedly contrived because most parents could find some reason for preferring that one child or the other receive the present, perhaps by considering the recent history of presents received, special interests, current behavior, etc. However, if the problem is as posed then the rules would indicate that there should be no difference between the probabilistic mixture and giving the present to either child.

Let us ask whether it is important to the mother that she inform the children about the basis on which the choice was made. For example, does the mother say, "John and Mary, I have a present here, and I am going to flip a coin to see who gets it", or does she flip the coin in private and appear before the children with the simple statement, "Mary, I have a present for you" (in the case where Mary has won the coin toss in absentia)? In other words, are we really talking about the prospects "John gets present" and "Mary gets present", or the prospects of each child’s getting the present with knowledge that the present was received as the result of a coin toss?
If the mother is trying to send a signal about impartiality by informing the children of the coin toss, then it appears that the prospects are not as originally described. In other words, the prospects are not only the original two of each child’s getting the present, but two more where each child gets the present with the knowledge that the receipt of the present was the result of the toss of a coin and not of a conscious decision on the part of their mother. If the mother is truly indifferent regarding all four prospects, then it should not make any difference whether she gives the present to one of the children or flips a coin. If, however, she would prefer the prospects where the children know that the receipt was the result of a coin toss and not of her conscious decision, then both these prospects will be at a higher level than the other two and the preferred strategy will be the one of tossing the coin. Thus in this case, there is no reason to adopt any rules other than our standard ones to govern reasonable behavior.

As you might expect, the discussions of equity are not primarily concerned about giving a present to children, but about making decisions where some calamity or benefit may fall upon certain people. The notion is that having every person have an equal chance of receiving this distinction is somehow more “equitable” than giving it to any person, even though the decision maker is indifferent to who receives it. We could resolve this problem in the same way as we did the decision of the mother by considering the prospects as decidedly different. However, another issue arises. That is equity, not from the point of view of the giver, but from the point of view of the receiver.

Here we must distinguish two main cases, one in which all the participants in the process have agreed to the process and a second in which this is not so. In the case of voluntary agreement to the process, there is no notion of equity, equal probability, or fairness required since the action was entirely voluntary. Thus, in forming a new company various participants may receive different amounts of stock in the enterprise because their abilities are different. Once the deal has been voluntarily entered into, no one can claim that any particular person’s share was inequitable—because he or she accepted the deal when they could have rejected it. We, therefore, have no need of the notion of equity in voluntary arrangements.

In the case of involuntary arrangements, we distinguish between receiving a benefit and a harm. If what is to be given is a benefit owned by the giver that anyone would be happy to receive then the only decision that matters is that of the giver, and whatever he or she thinks is fair must be fair no matter how special the tastes and preferences of that giver are.
For example, a billionaire walking in the street might give $10 million to a passer-by because she reminds him of his mother. Is that fair? Who knows? She certainly would be very happy to get the $10 million. Another example is that of triage, where limited medical facilities are available following a disaster. The providers of charitable medical benefits may allocate them as they wish.

The challenging case is the one where calamity is being visited upon people without their consent. Here the notion that it is more equitable to have the harm fall on each person with equal probability has to be a perversion of the word equitable. Imagine a terrorist holding 100 hostages who drew the name of the one to be executed from an urn rather than by simply walking up to a person and shooting him or her. Would we praise this terrorist for being so equitable? There is no equitable way to coerce innocent people.

In summary, the issue of equity does not impugn the normative rules in any case of ethical individual decision making. The use of the concept of equity is potentially unconscionable in group decision making where some harm is to be delivered unto the innocent. The notion of equity challenges not the normative rules, but rather their ethical application.

**The Role of the Normative Theory in Practice**

Having just touched on the realm of practice, it is appropriate to comment on two issues that are often discussed. The first is whether the normative theory is "prescriptive;" the second is whether it is applicable to group decision making.

**Normative/Prescriptive**

It is clear from my use of the word *normative* that I am using it in the sense of rules that *should* govern decision making. Some authors have chosen to use the word normative in another sense, and to introduce a new word *prescriptive*. They use normative to mean an ideal description of individual decision making that should not necessarily be followed in actual practice. They use the word prescriptive to refer to the decision process to be recommended to a decision maker, even if the normative rules are violated by this process.

It seems to me that what these authors refer to as prescriptive rules, I would refer to as the approximations that are appropriate when applying
the norms in practice. These approximations are not mistakes in the sense that they are violations of the norms of decision making, but are rather the interpretations required to apply the norms sensibly in the world.

To illustrate this distinction, consider a group of people deciding how to share the cost of a meal. The principle of “each should pay for what he has ordered” and the norms of arithmetic would constitute a decision process that could clearly be carried out by the diners. However, it is often the case that the diners regard the cost of going through the calculations as more onerous than any error that might arise by using the simpler process of dividing the bill by the number of diners, perhaps with minor adjustments made for people who ate lightly or heavily. Note that this procedure does not reject the norms of arithmetic, but rather accepts the higher decision rule that you should not spend more resources on a decision process than the results of that process are worth to you. Note, for example, that if the exact bills could be magically delivered to the individuals then there would be no need for the approximation.

When I coined the term decision analysis, I defined it as applied decision theory. Decision analysis is the engineering use of the norms of decision theory (the old time religion) in the practical world. The only time it would be worth taking a risk of violating the norms would be when the cost of assuring that they were satisfied was more than the cost of the potential violation, as in the meal example. In other words, if analysis were free and instantaneous, I can see no excuse for violating the norms in any decision, and therefore no reason to think of decision analysis or decision engineering as any different from any other engineering discipline that (by its nature) requires approximation. Consequently, I find no need to use the word prescriptive instead of normative to describe any aspect of practice.

Individual and Group Decision Making

Another source of confusion among many practitioners is whether the rules for decision making that we have discussed are applicable to group as well as individual decisions. A group could voluntarily accept there rules to govern group decision making. In this case, we would be using the decision rules as a metamodel or metaprocess for decision making by the group. Subgroups could be appointed to formulate the basis elements of alternatives, information, and preferences, and another subgroup could carry out the formal analysis to find the best decision. I once thought this would be an excellent way to carry out the process of
decision making in society (Howard, 1975), but I changed my mind because of the inevitable coercive nature of government. For private organizations it might still make a useful model.

However, one would have to be, professionally, extremely humble in applying such a metamodel, even in a private organization. We know from the seminal work of Arrow that there is no group decision process except dictatorship that satisfies a few simple requirements that we would place on any sensible decision process. The problems of gaming, misrepresentation, agenda manipulation, may all make sense to certain individuals in the group. Some people may be motivated to be deceptive about their representations of the basis elements. (Note that for an individual decision maker acting in self-interest, there is no advantage to any of the gaming tactics that may appear advantageous in a group.)

Consequently, in a group decision process, I would be very reticent to offer a warranty on the quality of decision making that applied to more than the evaluation of the given basis. Note that this is very different from having a specified decision maker who has personally accepted all the basis elements as appropriate for the organization. Indeed, this is the type of situation in which I feel most comfortable when conducting a decision analysis, for I can use effective procedures to guide the analysis.

To analyze an important decision, I and my closest colleagues at Strategic Decisions Group advise that the decision maker appoint two major participants, a content certifier and a process certifier. The content certifier, usually a senior member of the client organization, is responsible to the decision maker for the content of the decision basis. The process certifier, a professional consultant, is responsible to the decision-maker for the elicitation of all basis elements, for the structure of the basis, and for the execution of the analysis.

To ensure the integrity of the decision process, we require that each assessment in the decision basis be accompanied by a “pedigree.” In addition to the names of the content and process certifiers, the pedigree specifies the person responsible for the judgment represented in this particular assessment, the author, and the person responsible for conducting the elicitation process in which it was assessed—the analyst. In the case of a probability assessment, the pedigree contains a general description of the distinction being assessed and the clarity test definition of the distinction. The pedigree also records any notes regarding the general assumptions that are being made in the assessment and a list of the people who participate in the discussion. Then, if, at some later time, the decision maker asks, “Who said that the probability of technical success was 0.2?”, we can without delay present the whole story for the
decision maker's approval. A common comment is, "Well, I understand what was being assessed. Frank (the author) is our expert on this subject and as long as John and Mary (participants) were parties to the discussion, I see no reason to question it." When all assessments in the basis, whether of alternatives, information, or values, can be so scrutinized and evaluated, then I can give the strong warranty both that I have accurately represented the basis and that the course of action recommended is the one consistent with the basis.

When it comes to applying the individual decision-analysis paradigm, not only in a group situation, but in a government situation, then my position moves on from one of being professionally humble to one of being ethically repelled. To put the case as briefly as possible, a government action is inherently coercive, both in the way the funds for it will be raised and in the way the law will be used to govern the behavior of peaceful individuals. If the analysis is performed in terms of stakeholders who must be satisfied, I ask who appointed them to take and spend my money. I ask in any decision process how many votes does the taxpayer or other victim of the democratic process get in this assemblage of special interest groups. While I do not doubt the good faith of many who choose to participate in the process, to do so is to be an accomplice in coercion. In my view, the only ethically proper role in participating in government action would be in redressing the effects of previous government coercion. However, even in this role the analyst may find that he or she has become a party to further coercion.

**Conclusion**

Let me summarize my position on axiomatizations of decision theory that are at variance with the decision composite I have taken as fundamental in this chapter. If these alternate decision theories are to be used descriptively, then no one can object to them. Their test will be whether they describe human decision making. It might be very useful to have more powerful descriptive theories than we have at present. Such theories could provide a new foundation for results in fields from descriptive economics to consumer marketing.

What I take issue with is any suggestion that revisionist approaches should be used normatively to guide individual human decision making, and, in particular, decision analysis. I know of no scheme other than the one that I have presented that meets all the desiderata I have described in this chapter. I am personally unwilling to sacrifice any of these properties
for any benefit that has yet been offered. My concern is that the advocacy of these revisionist approaches for normative decision making may do harm by appearing to make inferior thought acceptable. In summary, "the old time religion is good enough for me." We need more evangelists, not alternate bibles.

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Note

1. Notation: \( \{ \cdot | \cdot \} = \text{Prob}(\cdot | \cdot) \); \( \langle \cdot | \cdot \rangle = \text{Exp}(\cdot | \cdot) \); \& is initial state of information (Howard, 1966).

References